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STATISTICAL PROPERTIES OF ALLOCATION AVERAGES

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STATISTICAL PROPERTIES OF ALLOCATION AVERAGES

BACKGROUND AND PURPOSE

In an earlier paper, Brogden[1] established that estimates of job performance, if derived from a battery of predictor variables according to least squares analysis under a model for linear regression, will place men in jobs in the most efficient way possible for the given predictors. For an optimal assignment based on such performance estimates for an infinite number of individuals, the average performance over the jobs to which men are assigned, if computed from the least squares performance estimates, will equal the average performance after assignment computed directly from measures of actual performance.

The purpose of this paper is to describe a simulation experiment which was performed to verify empirically Brogden's result concerning the unbiased nature of allocation averages. The theoretical proof for equality of allocation averages, computed from measures of actual performance and from least squares estimates of this performance, requires that optimal assignment be performed over least squares performance estimates for an infinite number of individuals. In the many simulation experiments conducted in BESRL's Statistical Research and Analysis Division, the allocation average computed from performance estimates, rather than from actual performance values, is often the basic statistic from which other experimental results are evaluated. The number of observations on which these results are based is, however, far from infinite. There was thus need to demonstrate that the two kinds of allocation average remain essentially equal when based on relatively small numbers of observations, as is typical of actual experiments.

It is conceivable that 1) when the number of observations, n, is small, allocation averages based on least squares estimates of performance are biased relative to allocation averages representative of measures of actual performance; 2) this bias, if it exists, occurs in a certain direction--allocation averages based on the estimates may be consistently larger than averages based on performance values for comparable numbers of observations; and 3) the size of this bias changes as a function of the number of observations. If any of these conceivable effects do in fact exist, then any experimental results previously obtained which involve optimal assignment of performance estimates and the allocation average statistic should be examined, and future experiments which involve the allocation average statistic will have to be corrected for the effect of the number of observations.

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[1] Brogden, H. E. Least squares estimates and optimal classification. Psychometrika. 20: 3, 1955.

Using notation similar to Brogden's, the result which needs to be verified experimentally is

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \hat{y}_{ij}\hat{a}_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{m} y_{ij}\hat{a}_{ij} \tag{1}$$

as n changes from small to large:

y_{ij} represents the performance of individual i in job j

$\hat{y}_{ij}$ represents the least squares estimate of job performance

a_{ij} represents an element of any allocation matrix A with n rows (corresponding to individuals) and m columns (corresponding to jobs). When individual i is assigned to job j, $a_{ij} = 1$; otherwise, $a_{ij} = 0$. Each individual is assigned to one job; i.e., $\sum_{j=1}^{m} a_{ij} = 1$; the number of individuals required for each job is q_j and $\sum_{i=1}^{n} a_{ij} = q_j$. The sum of the quotas is, of course, the total number of individuals: $\sum_{j=1}^{m} q_j = n$. An additional requirement on the allocation matrix is that the elements be constructed solely with respect to the least squares performance estimates.

$\hat{a}_{ij}$ represents an element of a particular allocation matrix $\hat{A}$ constructed so that

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \hat{y}_{ij}\hat{a}_{ij} \geq \sum_{i=1}^{n} \sum_{j=1}^{m} \hat{y}_{ij}a_{ij} .$$

That is, for a given set of performance estimates, $\sum_{i=1}^{n} \sum_{j=1}^{m} \hat{y}_{ij}\hat{a}_{ij}$ is the optimal allocation sum in the sense that no other arrangement of elements in $\hat{A}$, under condition that the requirements on any A are satisfied, will yield a higher sum.

Elements of the optimal allocation matrix $\hat{A}$ are constructed from a set of constants k_j, $j = 1,2,\ldots,m$, such that each individual i is assigned to the job j for which $(\hat{y}_{ij} + k_j)$ is highest; i.e., $\hat{a}_{ij}$ is set to 1. Thus, by definition,

$$\sum_{i=1}^{n} \sum_{j=1}^{m} (\hat{y}_{ij} + k_j)\hat{a}_{ij} \geq \sum_{i=1}^{n} \sum_{j=1}^{m} (\hat{y}_{ij} + k_j)a_{ij}$$

or, equivalently,

$$\sum_{j=1}^{m} \left(\sum_{i=1}^{n} \hat{y}_{ij}\hat{a}_{ij} + k_j q_j\right) \geq \sum_{j=1}^{m} \left(\sum_{i=1}^{n} \hat{y}_{ij} a_{ij} + k_j q_j\right)$$

since

$$\sum_{i=1}^{n} \hat{a}_{ij} = \sum_{i=1}^{n} a_{ij} = q_j .$$

Therefore,

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \hat{y}_{ij}\hat{a}_{ij} \geq \sum_{i=1}^{n} \sum_{j=1}^{m} \hat{y}_{ij} a_{ij} .$$

To prove equation (1), that $\sum_{i=1}^{n} \sum_{j=1}^{m} \hat{y}_{ij}\hat{a}_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{m} y_{ij}\hat{a}_{ij}$ as $n \to \infty$, Brogden considers the n individuals as being divided into subsets of individuals that have "identical patterns" on the battery of predictor tests from which the performance estimates are determined. The addition of the allocation constants to the corresponding performance estimates will result in all individuals within such a subset being assigned to the same job; i.e., the $\hat{a}_{ij}$, $j = 1,2,\ldots,m$, are constant for the n^* individuals within the subset. Therefore,

$$\frac{1}{n^*}\sum_{j=1}^{m}\sum_{i=1}^{n^*}\hat{y}_{ij}\hat{a}_{ij} = \frac{1}{n^*}\sum_{j=1}^{m}\hat{a}_{ij}\sum_{i=1}^{n^*}\hat{y}_{ij}.$$

Least squares performance estimates $\hat{y}$ derived under assumption of a linear regression model are unbiased estimates of the population mean μ for the performance values, given the fixed predictor variables $\underline{X}$; i.e.,

$$E(\hat{y} \mid \underline{X}) = E(y \mid \underline{X}) = \mu(y \mid \underline{X}).$$

Expressed in terms of observations, with each subgroup representing a given set of values for the predictor variables,

$$\frac{1}{n^*}\sum_{i=1}^{n^*}\hat{y}_{ij} = \frac{1}{n^*}\sum_{i=1}^{n^*}y_{ij}$$

and

$$\sum_{j=1}^{m}\hat{a}_{ij}\sum_{i=1}^{n^*}\hat{y}_{ij} = \sum_{j=1}^{m}\hat{a}_{ij}\sum_{i=1}^{n^*}y_{ij}$$

or

$$\sum_{i=1}^{n^*}\sum_{j=1}^{m}\hat{a}_{ij}\hat{y}_{ij} = \sum_{i=1}^{n^*}\sum_{j=1}^{m}\hat{a}_{ij}y_{ij}$$

as $n^* \to \infty$.

Since this result holds for any subgroup, it also holds in summing over all individuals. Consequently, the equality in (1) is established.

DESIGN OF THE SIMULATION EXPERIMENT

Assuming a model based on linear regression,

$$\underline{Y} = \underline{X}\hat{B} + \underline{E} = \hat{\underline{Y}} + \underline{E},$$

$\underline{Y}$ is a vector of random elements representing observations for an individual on m performance measures; $\underline{X}$ is a vector of fixed variables representing an individual set of scores on k predictor tests; $\hat{B}$ is a k x m matrix of random variables which are least squares estimates of the population regression coefficients B; $\underline{E}$ is a random vector representing measurement error; and $\hat{\underline{Y}} = \underline{X}\hat{B}$. Assume that $\underline{E}$ has the m variate normal distribution with mean vector $\underline{\emptyset}$ and covariance matrix C_E; then, letting Y and X represent matrices of n vector observations,

$$\hat{B} = (X'X)^{-1}X'Y \quad \text{and} \quad \hat{C}_E = \frac{Y'(I - X(X'X)^{-1}X)Y}{n - m}$$

are unbiased estimates of B and C_E; $\hat{B}$ and $\hat{C}_E$ are independent; each column vector $\underline{\hat{B}}_j$, $j = 1,2,\ldots,k$, and, consequently, $\hat{\underline{Y}}$ has an m variate normal distribution.

In addition, if $E(\hat{\underline{Y}}) = \underline{0}$,

$$\text{Cov}(\underline{Y}) = E(\underline{Y}'\underline{Y}) = C_Y$$

$$\text{Cov}(\hat{\underline{Y}}) = E(\hat{\underline{Y}}'\hat{\underline{Y}}) = E(\hat{B}'X'X\hat{B}) = E(Y'X(X'X)^{-1}X'Y) = C_{\hat{Y}}$$

$$\text{Cov}(\underline{Y},\hat{\underline{Y}}) = E(\underline{Y}'\hat{\underline{Y}}) = E(\underline{Y}'X\hat{B}) = E(\underline{Y}'X(X'X)^{-1}X'\underline{Y}) = C_{\hat{Y}}.$$

Then the 1 x (m + m) partitioned random vector $[\underline{Y}:\hat{\underline{Y}}]$ has expected mean vector

$[\underline{0} : \underline{0}]$ and covariance matrix $\begin{bmatrix} C_Y & C_{\hat{Y}} \\ C_{\hat{Y}} & C_{\hat{Y}} \end{bmatrix}$

Estimates of C_Y and $C_{\hat{Y}}$ are

$$\hat{C}_Y = \frac{Y'Y}{n} = C_{YY}$$

and

$$\hat{C}_{\hat{Y}} = \frac{Y'X(X'X)^{-1}X'Y}{n} = C_{YX}C_{XX}^{-1}C_{XY}$$

Suppose $\left[\underline{Y}_i : \hat{\underline{Y}}_i\right]$ is to be any vector of observed values of the random variables $[\underline{\mathcal{Y}}_i : \hat{\underline{\mathcal{Y}}}_i]$. Let $\underline{V}_i$ represent an observation in a random sample drawn from the m + m variate normal distribution $N(\underline{0}, I)$; i.e., $\underline{\mathcal{V}}_i$ is a 1 x 2m vector of independent random normal deviates.

$$\text{Let } T = C_{\hat{Y}}^{\frac{1}{2}} = \begin{bmatrix} C_{YY} & C_{YX}C_{XX}^{-1}C_{XY} \\ C_{YX}C_{XX}^{-1}C_{XY} & C_{YX}C_{XX}^{-1}C_{XY} \end{bmatrix}^{\frac{1}{2}}$$

$$\text{Then } \left[\underline{Y}_i : \hat{\underline{Y}}_i\right] = \underline{V}_i\, T$$

$$\text{and } \frac{1}{n}\sum_{i=1}^{n}\left[\underline{Y}_i : \hat{\underline{Y}}_i\right]' \left[\underline{Y}_i : \hat{\underline{Y}}_i\right] = \begin{bmatrix} C_Y & C_{\hat{Y}} \\ C_{\hat{Y}} & C_{\hat{Y}} \end{bmatrix} \text{ as } n \to \infty. \tag{2}$$

The $\hat{\underline{Y}}_i$, for i = 1, 2,...,n, are suitable values to represent least squares performance estimates over which optimal assignment is to be performed. The $\underline{Y}_i$, for the same i = 1,2,...,n, represent the measures of actual performance which correspond to the predicted values. If $\hat{A} = \left\{\hat{a}_{ij}\right\}$ is the n x m optimal allocation matrix constructed with respect to the n x m performance estimates $\hat{Y} = \left\{\hat{y}_{ij}\right\}$; i.e.,

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\hat{y}_{ij}\hat{a}_{ij} \geq \sum_{i=1}^{n}\sum_{j=1}^{m}\hat{y}_{ij}a_{ij}$$

where $A = \left\{a_{ij}\right\}$ is any other allocation matrix which meets the required restrictions. Then

$$\hat{z} = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{m}\hat{y}_{ij}\hat{a}_{ij}$$

and $$z = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{m} y_{ij} \hat{a}_{ij}$$

provide suitable observations for the empirical test of the equality in (1).

Two series of simulations were performed. In one series, the allocation averages z and $\hat{z}$ were based on 100 independent samples, s, each with 100 individuals, n. In the other series, the sample size was 1000 for a total of 10 samples. The same sequence of random numbers was used to generate both series of allocation averages, however, so that the total number of entities generated ($10{,}000 = n \times s = 100 \times 100 = 10 \times 1000$, even though subdivided to form different sized samples) represented observations from the same individuals.

Observed values for $C_{\hat{Y}}$ are presented in Table 1. The value in each diagonal element of C_{yy}, the criterion covariance matrix, was 1.00; off-diagonal elements were set to 0.45. C_{XY}, the covariance matrix between predictor and criterion variables, was formed from Table A-1, TRN 163[2], rows AE, EL, GM, MM, CL, GT and columns VE, AR, PA, MA, ACS, SM, AI, ECI, CI, GIT. C_{XX}, the predictor covariance matrix, contained 1.00's in diagonal elements; in the off-diagonal elements were values for the AE, EL, GM, MM, CL, GT variables of Table A-I, TRN 126.[3]

The number of job categories was chosen to be 6; For each sample, an $n \times 6$ matrix of performance estimates was constructed from the second six elements, the $\hat{\underline{Y}}_i$, of the partitioned vectors $[\underline{Y}_i \vdots \hat{\underline{Y}}_i]$, $i = 1,2,\ldots,n$, generated as shown in (2). Optimal assignment was then performed from this matrix, with nearly uniform quotas specified for each job; i.e.,

$$q_j = 16 \text{ or } 17 \text{ for } n = \sum_{j=1}^{6} q_j = 100 \text{ and } q_j = 166 \text{ or } 167 \text{ for } n = \sum_{j=1}^{6} q_j = 1000.$$

The linear programming procedure used to obtain optimal assignment was the Hungarian solution[4] to the transportation problem.

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[2] Sorenson, Richard C. Optimal allocation of enlisted men--Full regression equations vs aptitude area scores. BESRL Technical Research Note 163. November 1965.

[3] Katz, Aaron. Prediction of success in automatic data processing programming course. BESRL Technical Research Note 126. October 1962.

[4] Kuhn, H. W. The Hungarian method for the assignment problem. Naval Research Logistic Quarterly. 2: 1, 1955.

Using the same optimal assignment solution and the same sample of n partitioned vectors $[\underline{Y}_i \; \vdots \; \underline{\hat{Y}}_i]$ generated before assignment, allocation averages based on performance estimates, the $\underline{\hat{Y}}_i$, and allocation averages based on criterion values, the $\underline{Y}_i$, were computed to yield experimental observations $\hat{z}_t$ and z_t for sample t, t = 1,2,...,s.

RESULTS

ALLOCATION AVERAGES BASED ON PERFORMANCE MEASURES AND ESTIMATES

Mean allocation averages simulated over 100 samples of n = 100 and 10 samples of n = 1000 are presented in Table 2. The mean allocation average over s samples for performance estimates is

$$\hat{z}. = \frac{1}{s}\sum_{t=1}^{s}\hat{z}_t = \frac{1}{s}\sum_{t=1}^{s}\left(\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{m}\hat{y}_{ijt}\,\hat{a}_{ijt}\right)$$

and the mean allocation average over s samples for performance measures is

$$z. = \frac{1}{s}\sum_{t=1}^{s}z_t = \frac{1}{s}\sum_{t=1}^{s}\left(\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{m}y_{ijt}\,\hat{a}_{ijt}\right) .$$

The null hypothesis of interest is: allocation averages based on performance estimates and measures of actual performance are equal; i.e.,

$$H_o:\ E(\hat{\tilde{\xi}}.) = E(\tilde{\xi}.) = \xi .$$

$\hat{\tilde{\xi}}$. and $\tilde{\xi}$. are random variables which are estimates of the population parameter ξ. Corresponding observed values are $\hat{z}$. and z..
Under H_o,

$$\frac{\hat{\tilde{\xi}}. - \tilde{\xi}.}{\left[\mathrm{Var}\,(\hat{\tilde{\xi}}.) + \mathrm{Var}\,(\tilde{\xi}.) - 2\mathrm{Cov}\,(\hat{\tilde{\xi}}\,.\,\tilde{\xi}.\right]^{\frac{1}{2}}}$$

is distributed as t with s - 1 degrees of freedom. The decision rule adopted at the .05 level of significance is: Reject H_o if the observed $t \geq t_{.05}(s-1)$; accept H_o otherwise. For $\hat{z}$. and z. based on 100 samples of n = 100, observed t = 0.12< $t_{.05}(99) = 1.98$. Therefore, accept H_o. For $\hat{z}$. and z. based on 10 samples of n = 1000, observed t = 0.33< $t_{.05}(9) = 2.26$. Therefore, accept H_o.

The empirical results based on 100 samples of 100 observations and on 10 samples of 1000 observations are consistent with the equality in (1) proved by Brogden for an infinite number of observations.

DISTRIBUTIONS OF MEASUREMENT ERROR

The question of bias in allocation averages based on least squares performance estimates after optimal assignment can be rephrased in terms of distributions of measurement error. In previous sections, two kinds of allocation averages were computed after optimal assignment. Comparison of measures of sample performance obtained before and after optimal assignment are presented below:

Specifically, the null hypothesis to be tested was that mean differences between performance values and least squares estimates of performance values are the same both before and after allocation. Since $y_{ij} - \hat{y}_{ij} = e_{ij}$, this hypothesis is equivalent to determining whether bias is introduced into the distribution of the error components as a result of optimal assignment.

The measurement error frequency distributions before and after assignment are presented in Tables 3 and 4. In Table 3, the error terms for samples of 100 individuals are summarized over the 100 samples. Table 4 is a summary of the error distributions for 10 samples of 1000 individuals.

Statistical tests were based on the mean errors computed, before assignment, over all jobs and all individuals within a sample and, after assignment, over all jobs to which individuals were assigned. The observed grand mean based on all samples of n = 100 was .0026 before allocation and -.0007 after allocation; corresponding values for samples of n = 1000 was .0022 before allocation and -.0020 after allocation. Under the null hypothesis of no mean differences, letting $e_{b\cdot}$ and $e_{a\cdot}$ represent mean errors obtained before and after allocation, respectively,

$$\frac{e_{b\cdot} - e_{a\cdot}}{\left[\mathrm{Var}(e_{b\cdot}) + \mathrm{Var}(e_{a\cdot}) - 2\,\mathrm{Cov}\,(e_{b\cdot} \cdot e_{a\cdot})\right]^{\frac{1}{2}}}$$

is distributed as t with s - 1 degrees of freedom. Note that, as in the previous section, the test statistic takes into account dependence between the means.

At the .05 level of significance, the decision rule is to reject H_o if the observed $t \geq t_{.05}(s-1)$; accept H_o otherwise. For the mean errors based on 100 samples of $n = 100$, observed $t = 0.12 < t_{.05}(99) = 1.98$. Therefore, accept H_o. For the mean errors based on 10 samples of $n = 1000$, observed $t = 0.33 < t_{.05}(9) = 2.26$. Therefore, accept H_o.

Thus, no evidence was observed in the data for reproducible change in the distributions of measurement error as a function of optimal allocation, either for samples with $n = 100$ or $n = 1000$. This result is, of course, not surprising. The expected value of both performance values and estimates of performance values is zero before assignment and therefore the expected value of the differences will be zero. The null hypothesis of no mean differences between performance values and estimates after optimal assignment having been accepted, all means examined in the present section should be zero, and significant differences would not be expected.

OPTIMAL ALLOCATION AS A FUNCTION OF SAMPLE SIZE

The main purpose of the study was to examine in detail any differences which might occur in allocation averages as a function of number of observations. To do this, allocation averages $\hat{z}$ and z were based on solutions for optimal assignment computed over samples of $n = 100$. At the same time, allocation averages were obtained which were based on the same 10,000 entities but differed because each optimal solution was computed over only 1000 men.

It is often of practical interest, especially when it is important to conserve computer space, whether the allocation average based on an optimal assignment solution computed over a total sample of n individuals will be greater than the mean of k allocation averages computed from the same sample divided into k groups of n/k individuals. It could be conjectured that sample performance will be appreciably increased if the size of the sample on which optimization is based is increased. Experimental results relating to this hypothesis were incidentally available, since they were required for the main purpose of the study.

As shown in Table 2, observed mean allocation averages based on samples of $n = 1000$ tended to be higher than averages obtained from the 100-individual samples. It was not possible to draw consistent conclusions, however, using the t-test for correlated means as a statistical criterion. When both the solution for optimal assignment and the allocation average were based on least squares performance estimates, the mean for samples of $n = 1000$ ($\hat{z}. = .3056$) was significantly greater than the mean for samples of $n = 100$ ($\hat{z}. = .3019$) at the .01 level; the observed t was 15.35, well over the critical value of 3.25 for 9 degrees of freedom. On the other hand, with the solution for optimal assignment being based on performance estimates but with the allocation average computed from criterion values, the mean for samples of $n = 1000$ ($z. = .3076$) was not significantly greater than the mean for samples of

$n = 100$ ($z. = .3026$). The critical value at the .05 level for 9 degrees of freedom is 2.26, which is higher than the observed t of 1.83.

SUMMARY

In a series of simulation experiments, optimal assignment was performed with respect to least squares performance estimates for finite samples. Two measures of allocation averages, one based on the least-squares performance estimates and the other on actual performance values, did not differ in statistical significance. Such results indicate lack of bias in either measure of optimal assignment and are consistent with Brogden's theoretical proof of equality of the two measures for infinite samples.

Table 1

VARIANCE-COVARIANCE MATRIX REPRESENTING POPULATION PARAMETERS FOR PERFORMANCE MEASURES AND PERFORMANCE ESTIMATES
(Symmetrical elements are omitted)

$$C = \begin{bmatrix} C_{YY}^{a} & C_{\hat{Y}Y}^{b} \\ C_{Y\hat{Y}}^{b} & C_{\hat{Y}\hat{Y}}^{c} \end{bmatrix} =$$

1.000						.2323					
.4500	1.0000					.2737	.4831				
.4500	.4500	1.0000				.2871	.4509	.4810			
.4500	.4500	.4500	1.000			.3128	.4861	.4799	.5580		
.4500	.4500	.4500	.4500	1.0000		.3423	.4733	.5230	.4907	.6745	
.4500	.4500	.4500	.4500	.4500	1.0000	.2903	.4727	.4598	.4562	.5456	.5173
.2323						.2323					
.2737	.4831					.2737	.4831				
.2871	.4509	.4810				.2871	.4509	.4810			
.3128	.4861	.4799	.5580			.3128	.4861	.4799	.5580		
.3423	.4733	.5230	.4907	.6745		.3423	.4733	.5230	.4907	.6745	
.2903	.4727	.4598	.4562	.5456	.5173	.2903	.4727	.4598	.4562	.5456	.5173

[a] C_{YY} = variance-covariance matrix for performance measures.

[b] $C_{Y\hat{Y}} = C_{\hat{Y}Y}$ = covariance matrix between performance measures and performance estimates.

[c] $C_{\hat{Y}\hat{Y}}$ = variance-covariance matrix for performance estimates.

Table 2

ALLOCATION AVERAGES BASED ON PERFORMANCE MEASURES

	100 Samples, N = 100				10 Samples, N=1000			
	Performance: Estimates		Measures		Performance: Estimates		Measures	
By Job	Average	S.D.	Average	S.D.	Average	S.D.	Average	S.D.
1	-.2811	.4347	-.2803	.9919	-.2909	.4286	-.2937	.9892
2	-.2105	.6136	.2062	.9538	.2070	.6103	.1932	.9566
3	.2147	.5914	.2189	.9056	.2188	.5911	.2306	.8923
4	.5697	.6166	.5734	.9043	.5676	.6121	.5742	.9025
5	.7867	.6379	.8064	.8451	.7942	.6392	.8062	.8508
6	.3414	.6044	.3211	.9121	.3405	.5958	.3385	.8849
Combined	.3019	.5863	.3026	.9207	.3056	.5836	.3076	.9140

Table 3

DISTRIBUTIONS OF MEASUREMENT ERROR BEFORE AND AFTER ASSIGNMENT FOR 100 SAMPLES OF 100 INDIVIDUALS

	Interval Midpoints								Total		
	-3.5	-2.5	-1.5	-.5	.5	1.5	2.5	3.5	Frequency	Mean	S.D.
Measurement Errors before Assignment											
Job 1	2	107	1153	3754	3681	1196	107	0	10000	.0046	.8812
Job 2	0	26	786	4129	4159	869	31	0	10000	.0176	.7288
Job 3	0	33	752	4196	4219	776	24	0	10000	.0030	.7126
Job 4	0	15	666	4317	4360	629	13	0	10000	-.0028	.6633
Job 5	0	1	442	4626	4561	368	2	0	10000	-.0118	.5694
Job 6	0	19	722	4307	4157	767	28	0	10000	.0051	.6996
								n =	60000		
									Grand Mean =	.0026	
Measurement Errors after Assignment											
Job 1	1	18	198	632	631	200	20	0	1700	-.0008	.8840
Job 2	0	6	148	696	698	149	3	0	1700	.0043	.7386
Job 3	0	4	132	701	740	116	7	0	1700	-.0042	.6987
Job 4	0	3	113	746	732	106	0	0	1700	-.0036	.6611
Job 5	0	0	77	731	738	54	0	0	1600	-.0197	.5773
Job 6	0	2	111	686	671	128	2	0	1600	.0196	.6933
								n =	10000		
									Grand Mean =	-.0044	

Table 4

DISTRIBUTIONS OF MEASUREMENT ERROR BEFORE AND AFTER ASSIGNMENT
FOR 10 SAMPLES OF 1000 INDIVIDUALS

	Interval Midpoints								Total		
	-3.5	-2.5	-1.5	-.5	.5	1.5	2.5	3.5	Frequency	Mean	S.D.
Measurement Errors before Assignment											
Job 1	2	107	1153	3754	3681	1196	107	0	10000	.0046	.8812
Job 2	0	26	786	4129	4159	869	31	0	10000	.0176	.7288
Job 3	0	33	752	4196	4219	776	24	0	10000	.0030	.7127
Job 4	0	15	666	4317	4360	629	13	0	10000	-.0028	.6633
Job 5	0	1	442	4626	4561	368	2	0	10000	-.0118	.5693
Job 6	0	19	722	4307	4147	777	28	0	10000	.0022	.6997
								n =	60000		
								Grand:	Mean =	.0022	
									S.D. =		.7152
Measurement Errors after Assignment											
Job 1	1	18	192	619	624	196	20	0	1670	.0028	.8832
Job 2	0	7	133	692	677	157	4	0	1670	.0138	.7398
Job 3	0	4	129	711	704	115	7	0	1670	-.0118	.7003
Job 4	0	3	108	744	714	101	0	0	1670	-.0065	.6576
Job 5	0	0	78	754	768	60	0	0	1660	-.0120	.5783
Job 6	0	2	111	743	680	123	1	0	1660	.0020	.6781
								n =	10000		
								Grand:	Mean =	.0020	
									S.D. =		.7107